REPORT RESUMES

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THE TEACHER, PROGRAMED MATERIALS, AND INSTRUCTIONAL INTERACTION, SUPPLEMENTAL MANUAL.
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A DESCRIPTION OF MATHEMATICS DRILL PROBLEMS AND ENRICHMENT ACTIVITIES HAS RESULTED IN A SOURCE MANUAL TO ASSIST TEACHERS OF PROGRAMED ALGEBRA IN PROVIDING FOR INDIVIDUAL DIFFERENCES AMONG STUDENTS. THE MATERIAL WAS INTENDED FOR ASSIGNMENT TO SELECTED STUDENTS WHO REQUIRE ADDITIONAL MATERIALS TO SATISFY VARIOUS ABILITIES AND TALENTS. IN ADDITION TO REFERENCE LISTS ON FILMS AND FILMSTRIPS, PRINTED BIBLIOGRAPHIES WERE PREPARED ON POPULAR EXPOSITIONS, HISTORY, BIOGRAPHY, TRADITIONAL MATHEMATICS, MODERN MATHEMATICS, SPECIAL TOPICS, MATHEMATICAL RECREATIONS, TEACHING AIDS, PROGRAMED INSTRUCTION, AND PROGRAMED ALGEBRA. OTHER REPORTS ON THIS PROJECT WERE ASSIGNED ACCESSION NUMBERS ED 003 198, ED 003 237, AND ED 003 238. (JM)

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"The Teacher, Programed Materials and Instructional Interaction"

SUPPLEMENTAL MANUAL

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COOPERATIVE RESEARCH PROJECT

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SUPPLEMENTAL MANUAL

INTRODUCTION

This manual has been prepared for the express purpose of assisting the teacher in providing for individual differences among the students. The two general areas covered are drill problems and enrichment activities. The material found in this manual is provided merely as a suggestion to indicate the type of work which might be considered and is by no means complete within itself. The teacher is encouraged to use this material and to revise the suggestions by additions or deletions depending on the demands of his particular situation. The use of both the drill and enrichment materials will enable the teacher to provide a method by which the course may be supplemented in order to meet the particular needs of the student. This material is not intended to be assigned to the group at various intervals during the programed course but rather to selected students who require additional materials to satisfy various abilities and talents. One of the primary functions of the teacher will be to identify these particular needs and then to differentiate the course to meet these needs.

SUPPLEMENTARY DRILL

The primary purpose of the drill material is to fixate certain principles and concepts. While the actual method in which the materials will be employed is left to the discretion of the teacher, there are two methods which have merit. The first is to have the drill materials available to all students who, upon completing a unit of study, feel a need for further practice before being tested on the unit. In this instance much of the responsibility would rest with the student. The second method would be to use this material after providing remedial instruction to a student who has demonstrated on a unit test an insufficient understanding of a specific concept. Both these methods may be used concurrently in order to provide a more comprehensive understanding of the materials. It is necessary that each student demonstrate a certain degree of proficiency before being allowed to continue on to the next unit of study. Again, it should be emphasized that the problems represent only a suggestion as to the type of drill to be provided for the student and definitely do not represent a complete set of materials. The teacher may further supplement the program by additional drill materials to insure competency on the part of the student.

SUPPLEMENTARY ENRICHMENT

Suggestions for enrichment activities have been listed primarily for those students who have demonstrated a high degree of proficiency in working. the programed materials. The topics and projects found in this section of the manual cover a wide spectrum of interest and are of varying degrees of difficulty. Some of these topics can be covered in a few days while others will require almost a complete semester's work. These materials are not designed to provide a means of additional work for the student, but rather as a method by which the opportunity is afforded the student to explore topics of interest. Included in the suggested enrichment work are biographies of famous mathematicians, selected topics of study, and creative activities. Each of the listed items in the selected topics is followed by a series of sub-topics and numbers that correspond to specific references to be found in the bibliography. Other topics and references may be selected either by the student or the teacher. In teaching conditions I, II, and III, the enrichment work would be given concurrently with the work in the program itself. In Conditions IV and V the enrichment work is reserved until the program work is first mastered and then the remainder of the course time may be assigned to enrichment materials. Any enrichment project which is performed by the student should be presented to the class wherever applicable.

PROGRAMED MATERIALS

An additional type of study may be undertaken by the student through the use of other programed materials. These present the opportunity for a student to further increase his knowledge and background in mathematics. A prospectus of these courses will provide a description of the course content and other pertinent information concerning the materials. In the situation where the student covers a complete course of additional work, a record should be made of this on his permanent record card and if possible a mark should be given to designate the degree of proficiency demonstrated by the student.

FILM AND FILMSTRIPS

The use of films and filmstrips can offer effectively improved mathematical instruction if the lesson is properly structured. The teacher may use films to present additional topics to the class or to provide variance in the method of instruction. Also, films and filmstrips could be used to aid an individual student in the understanding of a particular topic of enrichment work. The majority of these films can be obtained through the various film rental libraries or purchased at a nominal cost. A word of caution should be added to insure that the proper use of the film for instructional purpose is obtained. The films are only as effective as the structure in which they are employed and sufficient pre-planning should be carried out before they are utilized.

BIBLIOGRAPHY

The bibliography is designed to serve two purposes. One is to present a list of reference information in conjunction with the student enrichment material and the second purpose is to provide a reference source for the teachers. These books and articles contain many topics which are pertinent to the study of mathematics and are suggested as a means by which a teacher may further expand his knowledge in a particular subject.

CHAPTER TESTS AND FINAL EXAMINATIONS

CONTENTS -

This book of testing materials has been prepared for the assistance of those teachers participating in the Cooperative Research Project on "The Teacher, Programed Materials and Instructional Interaction" who will be using the SRA programed learning materials. These tests have been developed through the combined resources of both the publishers of the materials and the Project staff. The staff is particularly grateful to the Science Research Associates, Inc., publishers of the SRA materials, for permission to edit the publisher's tests in order to make them more compatible with the design of the Project.

The contents of this book include twenty-two Unit or Chapter Tests, six Cumulative reviews, and one Final examination. These materials are presented to the participating teachers in order to insure a uniform method of evaluating the various teaching conditions of the Project. Since the data obtained from these tests will serve as a primary source of information for measuring the outcomes of the Project, it is essential that these examinations be used in their entirety in the form in which they are herein presented. It should be understood that any teacher may at his discretion prepare and administer tests or examinations in addition to those found in this book. Copies of these additional tests must be forwarded to the Research Project Center where they will become a part of the permanent record of the Project.

TEST SCORING -

The Project will be concerned only with the raw scores obtained from the testing materials found in this book. To achieve a degree of uniformity within the test data received from the teachers it is necessary to restrict the method of scoring the tests to the following procedure. No partial credit will be given in computing the raw score. Each numbered question will be considered as a single response, and the student will receive credit only if the entire question is answered correctly regardless of the number of parts associated with the problem. The total raw score will be the number of correct responses. A record of these raw scores is to be returned to the Field Consultant at various intervals throughout the year. Also the test answer booklets will need to be returned to the Project Center for the purpose of an item analysis and other statistical data.

The method of obtaining a student's mark on a particular test for local school use is left to the discretion of the teacher. After obtaining the information needed for the Project a teacher may readjust the marking of the test in any manner so desired. The tests are so constructed that many students will not be able to finish in the allotted time and this should be taken into consideration when computing marks.

TIME LIMITS -

To secure uniform test results a time limit has been determined for each type of test. These time limits are as follows:

Unit Tests	35 minutes
Cumulative reviews	40 minutes
Final examination	45 minutes

It is essential that these time limits be closely adhered to in order that the outcomes of the Project will rest on a valid base. The Final examination is constructed for a forty-five minute period, but a supplementary teacher-constructed test may be added in order to fulfill individual requirements set by the school district.

Participating teachers are urged to review the test materials carefully in advance so that any questions pertaining to content or procedure may be discussed with the Field Consultant.

UNIT 1

Supplementary Drill (Chapters 1-4)

Chapter 1: Numbers and Numerals

No further supplementary problems should be necessary.

Chapter 2: Directed Numbers

EXPRESS EACH OF THE FOLLOWING BY MEANS OF A DIRECTED NUMBER.

		Answers
1.	A withdrawal of $25.	-25
2.	A deposit of $75.	+75
3.	Twenty degrees below freezing on a Centigrade scale (Freezing is 0°C)	-20
4.	A mountain height of 6380 feet above sea level.	+6380
5.	An ocean depth of 330 feet.	-330
6.	A seven-yard gain in football.	+7
7.	A walk of three miles east.	+3
8.	Receipts of $17.	+17
9.	Eight units to the left of zero on the number scale.	-8
10.	Twenty-two units to the right of zero on the number scale.	+22

FOR EACH PAIR OF DIRECTED NUMBERS STATE WHICH ONE IS LARGER.

		Answers			Answers
1.	+7, +8	+8	6.	-9/2, -1/3	-1/3
2.	+9/2, +1/3	+9/2	7.	0, +5	+5
3.	+4, -2	+4	8.	0, -2	0
4.	+2, -4	+2	9.	-99, 0	0
5.	-6, -5	-5	10.	+5/2, -5/2	+5/2

ADD THE FOLLOWING DIRECTED NUMBERS.

	Answers		Answers
1. (+3) + (+4) =	+7	6. (-3/2) + (-1/2) =	-2
2. (+1/2) + (+3/2) =	+2	7. (-6) + (0) =	-6
3. (+7) + (0) =	+7	8. (-1/2) + (0) =	-½
4. (0) + (+5/3) =	+5/3	9. (-8) + (-5) =	-13
5. (-2) + (-3) =	-5	10. (-5) + (-8) =	-13

Chapter 3: Absolute Value and Directed Numbers

TRUE OR FALSE.

1. $\lvert +8\rvert = +8$	T	6. $\lvert -3\rvert + \lvert -5\rvert = -8$	F
2. $\lvert -231\rvert = -231$	F	7. $\lvert 19\rvert + \lvert -30\rvert = -11$	F
3. $\lvert -48\rvert = 48$	T	8. $-\lvert -4\rvert = +4$	F
4. $\lvert +25/2\rvert = -25/2$	F	9. $\lvert -(-3)\rvert = +3$	T
5. $\lvert +3\rvert + \lvert +16\rvert = +19$	T	10. $-\lvert +3\rvert - \lvert -2\rvert = -5$	T

ADD.

1. $\begin{array}{r}+4\\ \underline{+9}\end{array}$	+13	2. $\begin{array}{r}-3\\ \underline{+6}\end{array}$	+3	3. $\begin{array}{r}-5\\ \underline{-4}\end{array}$	-9	4. $\begin{array}{r}1/5\\ \underline{-4/5}\end{array}$	-3/5

5. -9 + 6 + 7 +(-3) =	+1	8. +1/2 + 5/2 + (-7/2) + 9/2 =	4
6. +7 +(-5) + (-6) + 2 =	-2	9. 8 + (-8) =	0
7. +4 + (-8) + 12 + (-8) =	0	10. -.5 + (-10.5) + .4 + (-1.1) =	11.7

Chapter 4: Real Numbers

FOR THE FOLLOWING NUMBERS, STATE THE ADJECTIVE (s) WHICH APPLY.
(Integer, rational number, irrational number, real number.)

1. +3	rational, integer, real	6. $\sqrt{-4}$	none
2. -7	rational, integer, real	7. $-\sqrt{3}$	irrational, real
3. $\sqrt{7}$	irrational, real	8. +½	rational, real
4. $-\sqrt{9}$	rational, integer, real	9. 0/7	integer, rational, real
5. 0	rational, integer, real	10. -3/5	rational, real

UNIT 2

Supplementary Drill

(Chapters 5-6)

Chapter 5: Sets and Subsets
Chapter 6: Union and Intersection

MATCH COLUMN I WITH ITS EQUIVALENT IN COLUMN II.

I	II	Answers
1. $\{1, 9, 25, \ldots\}$	a. {whole numbers between 1 and 2}	1, h
2. $\{0\}$	b. {odd integers between 1 and 7 inclusive}	2, g
3. $\emptyset$	c. {multiples of 7}	3, a
4. $\{1, 3, 5, 7,\}$	d. {numbers each of which equals its square}	4, b
5. $\{2, 4, 6, 8\}$	e. {the sum of 24 and 12}	5, i
6. $\{0, 1\}$	f. $\{z, y, x\}$	6, d
7. $\{0, 7, 14, 21, \ldots\}$	g. {zero}	7, c
8. $\{36\}$	h. {squares of odd numbers}	8, e
9. $\{0, 4, 16, 36\}$	i. {even integers between 2 and 8 inclusive}	9, j
10. $\{45\}$	j. {squares of even integers}	10, k
11. $\{x, y, z\}$	k. {the product of 15 and 3}	11, f

STATE WHETHER EACH OF THE SETS IS FINITE OR INFINITE (OPTIONAL).

1. {the whole numbers between 4 and 11}	finite
2. {the whole numbers between 0 and 5, inclusive}	finite
3. {the vowels in the word algebra}	finite
4. {all natural numbers}	infinite
5. {the months of the year which have fewer than 31 days}	finite
6. {the squares of all even numbers}	infinite
7. {the hairs on your head}	finite
8. {all fractions with numerator of 1}	infinite
9. {all negative integers}	infinite
10. {all people in the world}	finite

LET $A = \{1, 3, 5\}$ $B = \{3, 5, 7, 9\}$ $C = \{4, 6, 8, 10\}$

FIND:	Answers		Answers
1. $A \cup B$	$\{1, 3, 5, 7, 9\}$	7. $(A \cup B) \cup C$	{1, 3, 5, 7, 9, 4, 6, 8 10
2. $A \cap B$	$\{3, 5\}$	8. $A \cup (B \cup C)$	{3, 5, 7, 9, 4, 6, 8, 1 1
3. $A \cup C$	$\{1, 3, 5, 4, 6, 8, 10\}$	9. $A \cap (B \cap C)$	$\emptyset$
4. $A \cap C$	$\emptyset$	10. $(A \cap B) \cap C$	$\emptyset$
5. $B \cup C$	$\{3, 5, 7, 9, 4, 6, 8, 10\}$	11. $A \cap (B \cup C)$	$\{3, 5\}$
6. $B \cap C$	$\emptyset$	12. $(A \cap B) \cup (A \cap C)$	$\{3, 5\}$

LET $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9,\}$ $B = \{3, 6, 9, 12, \ldots, 99\}$ C = {5, 10, 15, 20, ...

INSERT IN THE BLANK EITHER $\in$ or $\notin$

	Answers		Answers
1. 3 ____ A	$\in$	7. 8 ____ A	$\in$
2. 3 ____ B	$\in$	8. 8 ____ B	$\notin$
3. 3 ____ C	$\notin$	9. 8 ____ C	$\notin$
4. 15 ____ A	$\notin$	10. 300 ____ A	$\notin$
5. 15 ____ B	$\in$	11. 300 ____ B	$\notin$
6. 15 ____ C	$\in$	12. 300 ____ C	$\in$

UNIVERSE OF DISCOURSE (I) AND A COMPLEMENT ($\bar{A}$) MUST BE EXPLAINED.

LET $I = \{a, c, e, i, n, r, t\}$ $D = \{c, e, n, t\}$

$A = \{t, r, a, i, n\}$ $E = \{n, i, c, e\}$

$B = \{a, r, t\}$ $F = \{t, a, r\}$

$C = \{r, e, t, a, i, n\}$

FIND	Answers		Answers
1. $A' =$	$\{c, e\}$	6. $A \cap C =$	A
2. $B' =$	$\{c, e, i, n\}$	7. $A' \cup C' =$	$\{c, e\}$
3. $D \cap C =$	$\{e, n, t\}$	8. $A \cup A' =$	I
4. $(C \cap D) \cap E =$	$\{e, n\}$	9. $A \cap A' =$	$\emptyset$
5. $C \cap D =$	$\{e, n, t\}$	10. $D' \cap E' =$	$\{a, r\}$

USING THE SETS ABOVE, INSERT ONE OF THE FOLLOWING $\subset$, =, $\not\subset$ IN THE BLANK. IF NONE APPLIES WRITE "NONE."

	Answer
1. B ____ A	$\subset$
2. B ____ F	=
3. D ____ E	$\not\subset$
4. D ____ E	$\not\subset$
5. B ____ A	$\not\subset$

UNIT 3

Supplementary Drill (Chapters 7-9)

Chapter 7: Ordered Pairs and Relations

STATE THE DOMAIN AND RANGE OF EACH OF THE FOLLOWING WHENEVER POSSIBLE.

Problem	Answers
1. $A = \{(1, 2), (2, 3), (2, 4), (-1, 3)\}$	$D_A = \{1, 2, -1\}$ $R_A = \{2, 3, 4\}$
2. $B = \{(1, 2), (2, 1)\}$	$D_B = R_B = \{1, 2\}$
3. $C = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$	$D_C = \{1, 2, 3, 4\}$ $R_C = \{1\}$
4. $E = \{(0, 0), (0, -1), (0, -2), (0, 3)\}$	$D_E = \{0\}$ $R_E = \{0, -1, -2, 3\}$
5. $F = \{1, 2, 3, 4\}$	not a relation
6. $G = \{a, b\}$	not a relation
7. $F \times G$	$D_{F \times G} = \{1, 2, 3, 4\}$ $R_{F \times G} = \{a, b\}$
8. $H = \{(0, 0)\}$	$D_H = R_H = \{0\}$
9. $I = \{(0, 0), (1, -1), (2, -2), (3, -3), (4, -4), \ldots\}$	D_I = {natural numbers} R_I = {non-positive integers}
10. $J = \{(\frac{1}{2}, 1), (2/3, 4), (-3/5, 2)\}$	$D_J = \{1/2, 2/3, -3/5\}$ $R_J = \{1, 4, 2\}$

FIND THE CROSS PRODUCT OF THE FOLLOWING.	Answers
1. $A \times B$ if $A = \{1, 2, 3\}$, $B = \{a, b\}$	$\{(1, a),(1, b),(2, a),(2, b),(3, a),(3,$
2. $B \times A$ if $A = \{1, 2, 3\}$, $B = \{a, b\}$	$\{(a, 1),(a, 2),(a, 3),(b, 1),(b, 2),(b,$
3. $C \times D$ if $C = \{-1, 1\}$, $D = \{0\}$	$\{(-1, 0),(1, 0)\}$
4. $E \times E$ if $E = \{1, 2\}$	$\{(1.1),(1, 2),(2, 1),(2, 2)\}$
5. $F \times G$ if $F = \{(1, 2), (3, 4)\}$, $G = \{5, 6\}$	$\{((1, 2), 5), ((1, 2), 6) ,((3, 4), 5), ((3, 4), 6) \}$

Chapter 8: Functions

STATE WHICH OF THE FOLLOWING ARE FUNCTIONS.	Answer
1. $A = \{(1, 2),(3, 4)\}$	yes
2. $B = \{(1, 2),(3, 2)\}$	yes
3. B^*	no
4. A^*	yes

5. $C = \{(-1, 2), (2, -1)\}$	yes
6. C^*	yes
7. $D = \{(2, 3), (2, 4), (3, 4)\}$	no
8. D^*	no
9. $E = \{(1, 1), (2, 4), (3, 9), (4, 16), (5, 25), \ldots\}$	yes
10. E^*	yes

STATE WHICH OF THE FOLLOWING SETS ARE SIMILAR.

$A = \{1, 2, 3\}$; $B = \{a, b\}$; $C = \{d, e, f\}$; $D = \{3, 2, 1\}$; $E =$ {all natural numbers} ; $F =$ {all even integers} ; $G = \emptyset$; $H = \{0\}$ $I = \{1\}$ $J = \{4, a, 3, b\}$

Answer: $A \sim C \sim D$; $E \sim F$; $H \sim I$

Chapter 9: Inequalities, Graphs, and Relations

TRUE or FALSE.

Problem	Answer
1. $2 > 1$	T
2. $3 > -4$	T
3. $0 < 3$	T
4. $-5 > 2$	F
5. $-3 \leq -3$	T
6. $.0001 \geq -987$	T
7. $5 \leq 4$	F
8. $a < b$ means $b - a$ is negative	F
9. If $S = x: x \leq -2.5$ then $2 \in S$	F
10. If $T = \{(p, q): p = \frac{1}{2}q\}$ then $(1/8, 1/4) \in T$	T

COMPLETION.

	Answer
1. $(2, 3) \in \{(x, y) : y + \square\}$	1
2. $(\square, 8) \in \{(x, y): y = 2x\}$	4
3. $(4, 1) \in \{\square : y = x + 3\}$	(y, x)
4. $(-1, -1) \square \{(x, y): y = -x\}$	$\notin$
5. $(1, \square) \in \{(x, y) = y = x - 4\}$	-3

UNIT 4

Supplementary Drill (Chapters 10-12)

Chapter 10: Functions and their Converses

STATE THE CONVERSE OF EACH OF THE FOLLOWING FUNCTIONS AND TELL WHETHER OR NOT IT IS A FUNCTION:

	Answer
1. $A = \{(1, 2),(2, 2),(3, -1)\}$	no; $\{(2, 1),(2, 2),(-1, 3)\}$
2. $B = \{(2, -3),(-3, +2),(0, 0)\}$	yes; $\{(-3, 2),(2, 3),(0, 0)\}$
3. $C = \{(1, 1)\}$	yes; $\{(1, 1)\}$
4. $D = \{(-\frac{1}{2}, \frac{1}{2}),(-1/3, 1/3)\}$	yes; $\{1/2, -1/2),(1/3, -1/3)\}$
5. $E = \{(1, 1),(2, 1),(3, 1),(4, 1)\}$	no; $\{(1, 1),(1, 2),(1, 3),(1, 4)\}$

USING THE ABOVE SETS, ANSWER THE FOLLOWING:

	Answer
1. In set A, the image of 3 is _______	-1
2. In set A, the image of 2 is _______	2
3. In set B, the pre-image of -3 is _______	2
4. $D(-\frac{1}{2})$ = _______	$\frac{1}{2}$
5. In set C, the image and the _______ are the same.	pre-image
6. In set E, the image of any element is _______	1
7. Set A $\sim$ set ______	B
8. Set ______ = set C*	C
9. Set _____ contains the most elements	E
10. In set E the pre-image of 1 is	1, 2, 3, or 4

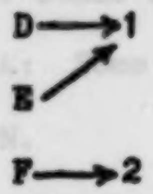
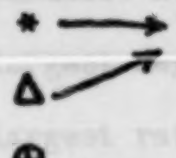
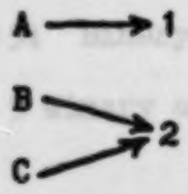
Chapter 11: Complements of Sets

COMPLETION:

	Answer
1. $\overline{A} = \{x: x \notin \square\}$	A
2. If $B = \{1, 2, 3\}$, $4 \in \square$	$\overline{\overline{B}}$
3. If $\bigcup$ = all natural numbers a and $C = \{1, 3, 5, 7, 9, \ldots\}$ $\overline{C} = \{\quad\}$	0, 2, 4, 6, 8, ...

4. If $\bigcup$ = {all integers} and D = {x: x is a non-negative integer} then $\overline{D}$ =	$\{0, -1, -2, -3, \dots\}$
5. If A = {x: x ≤ -2} then $\overline{A}$ =	$\{x: x > -2\}$

Chapter 12: Mappings

Problems	Answer
1. Mapping and ________ mean the same thing	function
2. If A = 0, 1 and B = -1, 2 one way of matching them one to one is ________	$0 \longrightarrow -1$ $1 \longrightarrow 2$ (others)
3. Is the following a one to one mapping?  Is it many to one?	no yes
4. Is the following a one to one mapping? A = *, Δ, ⊕ B = 1 	no, it is not even a mapping
5. List the images from problem 3.	1, 2
6. List the pre-images from problem 3.	D, E, F
7. Let F = {(1, 2),(2, 1),(3, 5)}. Is F a relation? A function? A mapping?	yes (all three)
8. If M and M* are functions, M* is the ________ of M.	inverse (converse)
9. Consider M, the following mapping.  Is the mapping one to one?	no
10. State the range and domain of M = $\{(1, 2),(3, 2),(7, -\frac{1}{2}),(0, 0)\}$	$D_M = \{1, 3, 7, 0\}$ $R_M = \{2, -2, -\frac{1}{2}, 0\}$

UNIT 5

Supplementary Drill (Chapters 13-15)

Chapter 13: Points and Real Numbers

TRUE or FALSE.	Answer
1. A one to one correspondence exists between the points of the number scale and the rational numbers.	F
2. A one to one correspondence exists between the points of the number scale and the irrational numbers.	F
3. A one to one correspondence exists between the points of the number scale and the real numbers.	T

Chapter 14: Some Order Properties

COMPLETION.	
1. The smallest positive integer is _______	+1
2. The largest positive integer is _______	does not exist
3. The smallest negative integer is _______	does not exist
4. The largest negative integer is _______	-1
5. The largest rational number is _______	does not exist
6. The smallest rational number is _______	does not exist
7. The largest negative number is _______	does not exist
8. The next largest number nearest to zero is _______	+1
9. The next largest number nearest to zero is _______	does not exist
10. Between any two rational numbers there are how many other rational numbers?	an infinite number

Chapter 15: Binary Operations

	Answer
1. Name 5 binary operations in mathematics	addition, subtraction, multiplication, division, cross product.

2. Given the table:

⊕	1	2	3	4
1	4	3	2	1
2	3	2	1	4
3	2	1	4	3
4	1	4	3	2

Perform 10 "circle additions."
eg. 1 ⊕ 1 = 4

UNIT 6

Supplementary Drill (Chapters 16-18)

Chapter 16: Closure

STATE WHICH OF THE FOLLOWING SETS ARE CLOSED UNDER THE GIVEN OPERATIONS. IF NOT CLOSED GIVE A COUNTER-EXAMPLE.

	Answer
1. A = {0, 1, 2, 3, 4} , addition	not closed
2. B = {0, 1} , multiplication	closed
3. C = {0} , addition	closed
4. D = {1} , subtraction	not closed 1-1 = 0, 0 ∉ {1}
5. E = {0, 2} , subtraction	not closed 0 - 2 = -2, -2 ∉ {0, 2}
6. F = {0, 1} , division	not closed is not defined
7. G = {0, 1, 2, 3, 4, 5, ...}, subtraction	not closed 4 - 5 = -1, -1 ∉ G
8. H = {0} , multiplication	closed
9. I = {1, 3, 5, 7, ...}, multiplication	closed
10. J = {1, 3, 5, 7, 9, 11, ...}, addition	not closed 3 + 5 = 8, 8 ∉ J
11. K = {★, Δ, δ} , *	not closed δ * δ = 𝄞, 𝄞 ∉ K

*	★	Δ	δ
★	Δ	δ	★
Δ	δ	★	Δ
δ	★	Δ	𝄞

Chapter 17: Identity Elements

	Answer
1. The identity element for addition is _____	0
2. The identity element for multiplication is _____	1
3. The identity element for division is _____	1
4. The identity element for subtraction is _____	0
5. Is i an identity element for ⊕ ? Justify your answer.	No ∈ ⊕ i = Δ (others)

⊕	δ	i	ϵ
δ	Δ	δ	ϵ
i	δ	i	Δ
ϵ	ϵ	Δ	δ

Chapter 18: Inverses

STATE THE INVERSE OF EACH OF THE FOLLOWING FOR THE INDICATED OPERATION.

	Answer
1. +6, addition	-6
2. -3, addition	+3
3. +2/3, multiplication	+3/2
4. -1/5, multiplication	-5
5. 3, division	3
6. -4, division	-4
7. 0, subtraction	0
8. 0, division	does not have an inverse under division.
9. $[7+(-3)]$, addition	-4

10.

*	a	b	c
a	b	c	a
b	c	a	b
c	a	b	c

	Answer
$\tilde{a}$ =	b
$\tilde{b}$ =	a
$\tilde{c}$ =	c

UNIT 7

Supplementary Drill (Chapters 19-23)

Chapter 19: Commutative Operations

STATE WHICH OF THE FOLLOWING ARE COMMUTATIVE OPERATIONS. IF NOT COMMUTATIVE, GIVE A COUNTER-EXAMPLE.

1. Multiplication	yes
2. Subtraction	no, $6 - 7 \neq 7 - 6$
3. Division	no, $6/2, \neq 2/6$
4. Union of sets	yes
5. Intersection of sets	yes
6. is a subset of	no, $A \subset B$ and $B \subset A$ are both true if, and only if $A = B$
7. cross product	no, $A \times B = B \times A$ if and only if $A = B$
8. Addition	yes

9.

*	a	b	c
a	b	c	a
b	c	a	b
c	a	b	c

yes

10.

π	α	β	γ
α	β	γ	α
β	γ	α	γ
γ	α	β	γ

no $\gamma \, \pi \, \beta = \beta$

$\beta \, \pi \, \gamma = \gamma$

Chapter 20: Subtraction of Directed Numbers

COMPLETION.	Answer		Answer
1. $16 - (8) =$	8	6. $(0, 0) \xrightarrow{-} \square$	0
2. $3 + (-4) =$	-1	7. $((5, 8), \square) \in -$	-3
3. $0 - (-5) =$	5	8. $((0, -2), \square) \in -$	2
4. $(8, 9) \xrightarrow{-} \square$	-1	9. $-(3, 2) \ \square \ -(6, 5)$	=
5. $(0, -5) \xrightarrow{-} \square$	5	10. $-(x, y) \ \square \ -(y, x)$	$\neq$ for all x and y

Chapter 21: Associative Operations

STATE WHICH OF THE FOLLOWING ARE ASSOCIATIVE OPERATIONS. IF NOT ASSOCIATIVE, GIVE A COUNTER-EXAMPLE.

	Answer
1. addition	yes
2. multiplication	yes
3. subtraction	no, $3-(4-5)=4$ $(3-4)-5=-6$
4. division	no, $(8 \div 4) \div 2 = 1$ $8 \div (4 \div 2) = 4$
5. union of sets	yes
6. intersection of sets	yes
7. * if $a * (b * c) = (a * b) * c$	yes
8. $\begin{array}{c\|c\|c} * & a & b \\ \hline a & b & a \\ \hline b & b & b \end{array}$	no, $(a * b) * a = b$ $a * (b * a) = a$

Chapter 22: Distributive Operations

STATE WHICH OF THE FOLLOWING ARE DISTRIBUTIVE OPERATIONS. IF NOT DISTRIBUTIVE, GIVE A COUNTER-EXAMPLE.

1. multiplication over addition	yes
2. addition over multiplication	no, $2 + (3 \cdot 4) = 2 + 12 = 14$ $(2+3) \cdot (2+4) = 5 \cdot 6 = 30$
3. multiplication over multiplication	no, $2 \cdot (3 \cdot 4) = 2 \cdot 12 = 24$ $(2 \cdot 3) \cdot (2 \cdot 4) = 6 \cdot 8 = 48$
4. Intersection over union	yes
5. Union over intersection	yes
6. Intersection over intersection	yes
7. $a * (b \gamma c) = (a * b) \gamma (a * c)$	yes
8. $a \Delta (b \int c) = (a \int b) \Delta (a \int c)$	no, form is incorrect

Chapter 23: Laws of Arithmetic - Review

STATE WHICH LAW OR PRINCIPLE OF MATHEMATICS IS BEING ILLUSTRATED..
i.e. Closure, identity element, inverse element, associative law, distributive law, commutative law.

	Answer
1. $\alpha + B = B + \alpha$	commutative law for addition
2. $\alpha \cdot (B \oplus r) = \alpha \cdot B \oplus \alpha \cdot r$	distributive law
3. $2, 3 \in R$ and $2 + 3 \in R$	closure addition of reals
4. $\alpha \cdot (B \cdot r) = (\alpha \cdot B) \cdot r$	associative law for multiplication
5. $*(2, 3) = *(3, 2)$	commutative law for *
6. $4 * 1 = 4$	1 is the identity element for *
7. $5, 6, \in R$ and $5/6 \in R$	closure division of reals

UNIT 8

Supplementary Drill (Chapters 24-27)

Chapter 24: Absolute Value As An Operation

TRUE or FALSE.

	Answer
1. If $a = 0$ then $\|a\| < 0$	false
2. If $r \in R$ then $\|r\| \in R$	true
3. The pre-image of π under $\| \|$ is π only.	false
4. If the domain of $\| \|$ is the set of all real numbers, the range of $\| \|$ is the set of non-negative real numbers.	true
5. $\| \|$ is a unitary operation because the domain of $\| \|$ is a set of single elements.	true

Chapter 25: Multiplication of Directed Numbers

	Answer		Answer
1. $(-4)(9 + 11) =$	-80	6. $\left[x(2, -3)\right] =$	-6
2. $(18 - 19)\,4 =$	-4	7. $-\left[x(3, 2)\right] =$	-6
3. $(5)(-6 - 3) =$	-45	8. $\left[x(1/8, 16)\right] =$	2
4. $0(-8 + 5) =$	0	9. $-\left[x(\frac{1}{2}, -3/5)\right] =$	$3/10$
5. $(-5)(-4) =$	20	10. $-\left[x(23/5, 0)\right] =$	0

Chapter 26: Equations, Pronumerals, Solutions Sets

STATE THE SOLUTION SET OF EACH OF THE FOLLOWING.

	Answer		Answer
1. $3 + x = 7$	$\{4\}$	6. $\Delta + 2 = \Delta$	$\emptyset$
2. $4 - * = 2$	$\{2\}$	7. $4 - ? = -11$	$\{-15\}$
3. $-8 + B = -2$	$\{6\}$	8. $1 + \alpha = -1$	$\{-2\}$
4. $\Delta - 8 = -8$	$\{0\}$	9. $y + 2 < 5$	$\{y: y < 3\}$
5. $5 - y = 3$	$\{2\}$	10. $\|t\| = 2$	$\{2, -2\}$

Chapter 27: Equations Involving Multiplication and Division

STATE THE SOLUTION SET OF EACH OF THE FOLLOWING:

1. $5x = 40$	$\{8\}$	4. $(B)(1/9) = -7$	$\{-63\}$	7. $11/m = -4$	$\{-11/4\}$
2. $3\Delta = -5$	$\{-5/3\}$	5. $7 \div r = 4$	$\{7/4\}$	8. $25/x \geq 5$ for $U = \{6, 7, 8, 9\}$	$\emptyset$
3. $\frac{1}{3}t = 4$	$\{12\}$	6. $\frac{5}{2}g = 2/5$	$\{4/25\}$	9. $\|3/g\| = 4$	$\{3/4, -3/4\}$

UNIT 9

Supplementary Drill (Chapters 28-31)

Chapter 28: Identities, Expressions, Terms

STATE WHICH OF THE FOLLOWING ARE IDENTITIES.

	Answer		Answer
1. $2x + x = 3x$	yes	4. $y/5 + y/5 = 2/5\ y$	yes
2. $t/3 + 5/3 = \frac{t + 5}{6}$	no	5. $2t = 8$	no
3. $\alpha + 2 + \alpha = 2$	no	6. $3 + 2 = 5$	yes

EVALUATE THE FOLLOWING:

	Answer		Answer
1. $\Delta - 3$ when $\Delta = 2$	-1	4. $5y + 7$ when $y = -1/5$	6
2. $s + 2/3$ when $s = -2/3$	0	5. $\frac{2 + 6r}{4}$ when $r = 1$	2
3. $-2t$ when $t = -4$	8	6. $8 \div (t + 1)$ when $t = -1$	not defined

STATE WHICH OF THE FOLLOWING ARE BASIC EQUATIONS:

	Answer		Answer
1. $x - 7 = 11$	no	4. $-s = 7$	no
2. $2y = 7$	no	5. $5 = q$	yes
3. $t = 5 - 1$	yes	6. $y/2 = 5$	no

Chapter 29: Expressions, Equations, and Identities with Two Pronumerals

EVALUATE THE FOLLOWING:

		Answer
1. $3a + 2b$	when $a = 2$, $b = 3$	12
2. $\psi \cdot \alpha$	when $\psi = 1/5$, $\alpha = 25$	5
3. $-2t/s$	when $t = 0$, $s = 8$	0
4. $8 - xy$	when $x = -2$, $y = 4$	0
5. $\beta \cdot \beta \cdot \beta$	when $\beta = 5$	125
6. $\frac{\alpha - \beta}{\alpha\beta}$	when $\alpha = 2/9$, $\beta = 2/9$	0

WRITE EQUIVALENT SIMPLIFIED EXPRESSIONS FOR EACH OF THE FOLLOWING:

	Answer		Answer
1. $8y - 3y$	$5y$	4. $1.8d - 3.6d$	$-1.8d$
2. $-3t + 2t$	$-t$	5. $.07\lambda - 1\lambda$	$-.93\lambda$
3. $-3\alpha - 8\alpha$	-11α	6. $3/2n - 3n/2$	0

7. $2\left[t - (3t + 7)\right]$	$-4t - 14$
8. $7\left[-5(r + s) + 3(s + 1)\right]$	$-35r = 14s + 21$
9. $-3\left[2(7\alpha + 5\beta - 6\gamma)\right]$	$-42\alpha - 30\beta + 36\gamma$
10. $5(\left[6(a + b) + c\right] + a)$	$35a + 30b + 5c$

Chapter 30: One Step Equations

FIND THE SOLUTION SET OF EACH OF THE FOLLOWING:

	Answer		Answer
1. $m - 3 = 5$	$\{8\}$	4. $-3 = \alpha - 5$	$\{2\}$
2. $2 = \Delta - 4$	$\{6\}$	5. $t/7 = 0$	$\{0\}$
3. $3t = -6$	$\{-2\}$	6. $-t + 5 = 0$	$\{5\}$

Chapter 31: More One Step Equations

FIND THE SOLUTION SET OF EACH OF THE FOLLOWING:

1. $7\alpha = 2$	$\{2/7\}$	6. $-8 = \frac{b}{-7}$	$\{56\}$
2. $8t = -3$	$\{-3/8\}$	7. $\alpha - 7 = -7$	$\{0\}$
3. $-32 = -4r$	$\{8\}$	8. $\alpha + \beta = \delta$ for β	$\{\delta - \alpha\}$
4. $st = -1$ for s	$\{-1/t\}, t \neq 0$	9. $-1 = -pt$ for t	$\left\{\frac{1}{p}\right\}, p \neq 0$
5. $-5c = 0$	$\{0\}$	10. $0 = \frac{7}{8}t$	$\{0\}$

UNIT 10

Supplementary Drill

(Chapters 32-34)

Chapter 32: Two Step Equations

FIND THE SOLUTION SET OF EACH OF THE FOLLOWING:

	Answer		Answer
1. $x + 2x = 6$	$\{2\}$	6. $-6 = 8 - (2/3)\triangle$	$\{21\}$
2. $4t - 5 = 7$	$\{3\}$	7. $t - rn = j$ for r	$\left\{\frac{t - j}{n}\right\}$
3. $at - ab = c$ for a	$\left\{\frac{c}{t - b}\right\}$	8. $f + x/m = d$ for x	$\{dm - fm\}$
4. $sr - a = p$ for s	$\left\{\frac{a + p}{r}\right\}$	9. $(-1/7)x + 1 = 1/7$	$\{6\}$
5. $k/2 - 8 = 0$	$\{16\}$	10. $8x = 13x$	$\{0\}$

Chapter 33: Linear Equations

FIND THE SOLUTION SET OF EACH OF THE FOLLOWING.	Answer
1. $(3/5)x = 6$	$\{10\}$
2. $6x - a = 11$ for x	$\left\{\frac{a + 11}{6}\right\}$
3. $8t - 3 = t$	$\{3/7\}$
4. $ax = -2x + g$	$\left\{\frac{g}{a + 2}\right\}$
5. $3 + 6k + 4 = 1 - 2k + 3 - k$	$\{-1/3\}$
6. $b - dx + e = ix + f + gx - j$ for x	$\left\{\frac{b + e - f + j}{i + g + d}\right\}$
7. $6 - 4(3f - 5) = 37 + (f - 7)$	$\{-4/13\}$
8. $5(x + 3) - 3(2 + x) + 5(x + 7) = 5(x + 3)$	$\{-29/2\}$
9. $\frac{3(m + 2)}{4} - \frac{5m}{4} = \frac{9 - m}{4} + \frac{2(m + 1)}{4}$	$\{-5/3\}$
10. $3 - 4(t - \frac{1}{2}) = 2/5(t + 7) + 5$	$\{-7/11\}$

Chapter 34: Fractional Equations

SOLVE.	Answer
1. $2x/3 = 4$	$\{6\}$
2. $16 = -8 + 3t/5$	$\{40\}$
3. $\dfrac{x}{x+2} = -3$	$\{-3/2\}$
4. $9 - 4/s = 5/s + 2$	$\{9/7\}$
5. $5 - \dfrac{3t-1}{2} = \dfrac{7t}{2} + \dfrac{4-t}{2}$	$\{7/9\}$
6. $\dfrac{gl - s}{l} - \dfrac{s}{l} + g = s$ for l	$\left\{\dfrac{2s}{2g - s}\right\}$
7. $3 = 2/5 - 4/m$	$\{-20/13\}$
8. $t/8 + 2 = 2$	$\{0\}$
9. $-5f/2 = 4 + 2/3$	$\{-28/15\}$
10. $\dfrac{y}{b-y} = \dfrac{ny}{b-y} + \dfrac{b}{a} + bn$ for y	$\left\{\dfrac{bb(1 + na)}{na(b-1) + (a+b)}\right\}$

UNIT 11

Supplementary Drill (Chapters 35-38)

Chapter 35: Word Problems

WRITE IN MATHEMATICAL SYMBOLS.	Answer
1. Three is equal to seven minus four	$3 = 7 - 4$
2. 12 minus one third of x is equal to x	$12 - 1/3x = x$
3. 5 times some number (g) is greater than or equal to minus one-half.	$5g \geq -\frac{1}{2}$
4. Some number (t) divided by the sum of 8 times the number and 2/3.	$\frac{t}{8t + 2/3}$
5. A number (s) divided by 3 times another number (q) is equal to 1/5 of the first number.	$\frac{s}{3q} = \frac{1}{5}s$

SOLVE THE FOLLOWING.	Answer
1. The sum of five times a certain number and three times that number is 72. Find the number.	9
2. One number is six times as large as another. If the smaller number is subtracted from the larger, the result is 35. Find the two numbers.	42, 7
3. Five days of a man's vacation were used sleeping. One-third of the remainder was exactly one week. How many days vacation did he have?	26
4. A father is five times as old as his son. In 4 years the sum of their ages will be 56 years. Find their present ages.	8 and 40
5. A test was given in which half of the people finished in 20 minutes. 5 more finished after 10 more minutes and the remaining 12 finished 10 minutes after those 5 people. How many people took the test? How long did the slowest people take to do the test?	34 people 40 minutes

Chapter 36: Properties of Inequalities

REFER BACK TO DRILL PROBLEMS FOR CHAPTER 9.

Chapter 37: Solving Inequalities	Answer
1. $x - 2 \geq 4$	$\{x: x \geq 6\}$
2. $y + 3 \leq 3$	$\{y: y \leq 0\}$

3. $2x + 5 > -6$	$\{x: x > -11/2\}$
4. $x - 3 \leq 3x + 7$	$\{x: x \geq -5\}$
5. $\frac{4}{5}x - 3 \leq 4$	$\{x: x \leq \frac{35}{4}\}$
6. $-\frac{1}{3}t + \frac{1}{6} \leq \frac{2}{6}$	$\{t: t \geq -\frac{1}{2}\}$
7. $3(m - 2) \leq 6(m + 2)$	$\{m: m \geq -6\}$
8. $a + \frac{a}{3} + \frac{a}{5} < 0$	$\{a: a < 0\}$
9. $x + 2 \leq 3 + x$	$\{x: x \in R\}$
10. $\frac{1}{5}(x - 7) > \frac{2}{3}(7 - x)$	$\{x: x > 7\}$

Chapter 38: Inequalities and Relations

COMPLETION.	Answer
1. If $U = \{-1, 0, 1\}$ then $> = \{\qquad\}$	(1, 0), (1, −1), (0, −1)
2. If $U = \{0, 1/2, 2/3\}$ then $< = \{\qquad\}$	(0, ½), (0, 2/3), (1/2, 2/3)
3. If $U = \{-1, 0, 1\}$ then $\leq = \{\qquad\}$	(−1, −1), (0, 0), (1, 1) (−1, 0), (−1, 1), (0, 1)
4. If $(a, b) \in >^*$, then a ___ b (> or <)	
5. If $U = \{x: x < 0\}$ then $>$ contains an __________ number of pairs belonging to the relation.	infinite

UNIT 12

Supplementary Drill

(Chapters 39-41)

Chapter 39: Properties of Absolute Value

TRUE or FALSE:

		Answer			Answer
1.	$\lvert 3+7\rvert = \lvert 3\rvert + \lvert 7\rvert$	T	6.	$\lvert(3)(-4)\rvert \leq \lvert 3\rvert\,\lvert -4\rvert$	T
2.	$\lvert 4+(-2)\rvert = \lvert 4\rvert + \lvert -2\rvert$	F	7.	$\lvert 3t\rvert = \lvert 3\rvert\,\lvert t\rvert$	T
3.	$\lvert -5+1\rvert \leq \lvert -5\rvert + \lvert 1\rvert$	T	8.	$\left\lvert \frac{3}{-5}\right\rvert > \frac{\lvert 3\rvert}{\lvert -5\rvert}$	F
4.	$\lvert -2-3\rvert = \lvert -2\rvert + \lvert -3\rvert$	T	9.	$\lvert 2/7\rvert = \frac{\lvert 2\rvert}{\lvert 7\rvert}$	T
5.	$\lvert 6+(-6)\rvert \leq \lvert 6\rvert - \lvert 6\rvert$	T	10.	$\lvert 4-3/5\rvert \leq \lvert 4\rvert + \left\lvert -\frac{3}{5}\right\rvert$	T

Chapter 40: Absolute Value Equations

SOLVE:

		Answer			Answer
1.	$\lvert x\rvert = 4$	$\{4, -4\}$	6.	$\lvert x\rvert - \lvert 2\rvert = 3$	$\{5, -5\}$
2.	$\lvert t\rvert = -3$	$\emptyset$	7.	$\frac{\lvert t\rvert}{3} = 7$	$\{21, -21\}$
3.	$3 - \lvert y\rvert = -7$	$\{10, -10\}$	8.	$\frac{\lvert g\rvert}{-2} = -6$	$\{12, -12\}$
4.	$\lvert w-2\rvert = 3$	$\{5, -1\}$	9.	$\frac{\lvert m-3\rvert}{5} = 0$	$\{3\}$
5.	$\lvert 3-s\rvert = -4$	$\emptyset$	10.	$\frac{2\lvert 4-t\rvert}{3} = 3$	$\{17/2, -1/2\}$

Chapter 41: Formulas

		Answer
1.	Solve $A = LW$ for L; for W	$L = \frac{A}{W}$; $W = \frac{A}{L}$
2.	Solve $P = 2(L + W)$ for L; for W	$L = \frac{1}{2}P - W$; $W = \frac{1}{2}P - L$
3.	Solve $I = prt$ for p; for r; for t	$p = \frac{I}{rt}$; $r = \frac{I}{pt}$; $t = \frac{I}{pr}$
4.	Solve $D = rt$ for r; for t	$r = \frac{D}{t}$; $t = \frac{D}{r}$
5.	Solve $A = \frac{1}{2}bh$ for b, for h	$b = \frac{2A}{h}$; $h = \frac{2A}{b}$
6.	If the area and base of a triangle are 40 square inches and 8 inches respectively, find in inches the altitude.	10 in.

7. If the perimeter of a rectangle is 30 inches and one side is 4 inches, find in inches the length of an adjacent side. 11

8. If a rocket travels 2 million miles in 8 days what is its rate in miles per hour? 10,416 2/3 m.p.h.

UNIT 13

Supplementary Drill (Chapter 42)

Chapter 42: Advanced Word Problems

1. A man has some coins consisting of nickels and dimes and he has twice as many nickels as he has dimes. Find the number of each kind of coin that he has if he has 36 coins altogether.
   24 nickels
   12 dimes

2. John is three times as old as his sister, Joy, and the sum of their ages is 28 years. Find the age of each.
   John is 21 yrs.
   Joy is 7 yrs.

3. The sum of the perimeters of two squares is 100 inches. If the perimeter of one square is four times the perimeter of the other, find the length of the side of each square.
   Side of the smaller is 5 in.
   Side of larger is 20 in.

4. Find two consecutive even integers such that five times the first added to three times the second gives a sum of 230. 28, 30

5. Frank working by himself can cut the lawn in 4 hours. His older brother Jim can do the same job in 2 hours. How long would it take Frank and Jim working together to cut the lawn? 4/3 hours

6. How many pounds of coffee worth \$1.00 per pound must be added to 40 lb. of coffee worth 70¢ per pound to make a mixture worth 90¢ a pound? 80 lbs.

7. If six is added to three-fourths of a certain number, the result is nine less than the number. Find the number. 60

8. A rectangle is 9 feet by 8 feet. Its area is three times the area of a rectangle twelve feet long. Find the width of the second rectangle. 2 feet

9. A man invests some money at 4% and twice as much money at 5%. If his annual income from these two investments is \$280, how much has he invested at each rate?
   \$2000 at 4%
   \$4000 at 5%

10. The numerator of a fraction is 3 less than the denominator. If 2 is subtracted from both numerator and denominator of the fraction, the new fraction will equal 3/4. What was the original fraction? 11/14

UNIT 14

Supplementary Drill (Chapters 43-46)

Chapter 43: Graphs of Relations

GRAPH EACH OF THE FOLLOWING ON A NUMBER SCALE. MAKE A SEPARATE SCALE FOR EACH ONE.

1. $\{x: x \geq 2\}$
2. $\{x: -2 \leq x < 3\}$
3. $\{x: x \text{ is an integer between } -1 \text{ and } 0\}$
4. $\{x: x > -3\}$
5. $\{x: x \text{ is a positive even integer less than } 7\}$

GRAPH EACH OF THE FOLLOWING RELATIONS ON A RECTANGULAR COORDINATE SYSTEM.

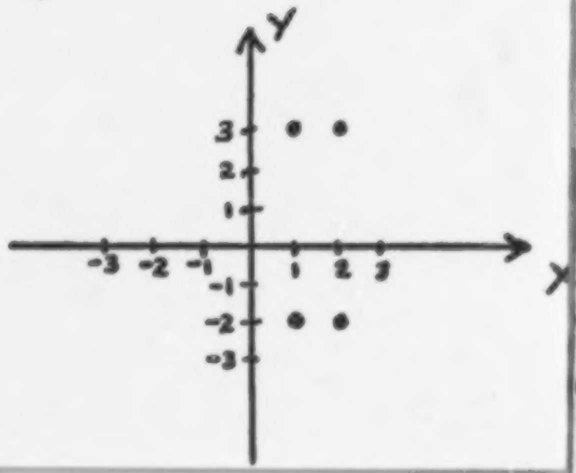
1. $\{(1, 2), (3, 4), (-1, -3), (-3, 5), (2, -3)\}$
2. $\{(0, 0), (0, 2), (0, -1), (0, 4)\}$
3. $\{(7, 0), (-3, 0), (\frac{1}{2}, 0)\}$
4. $\{(x, y): x > 0 \text{ and } y < 0\}$
5. $\{(x, y): x = 0 \text{ and } y \leq 0\}$

Chapter 44: Coordinate Plane

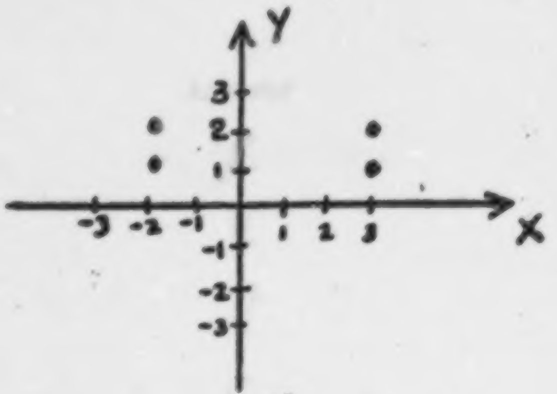
COMPLETION:	Answer
1. The set of x-coordinates of the relation $T = \{(1, -1), (2, -2), (3, 4), (4, 3)\}$ is ________	$\{1, 2, 3, 4\}$
2. The set of y-coordinates of the relation T is ________	$\{-1, -2, 4, 3\}$
3. The domain of T is ________	$\{1, 2, 3, 4\}$
4. The range of T is ________	$\{-1, -2, 4, 3\}$
5. The graph of a relation is a set of ________	points

GRAPH THE FOLLOWING:

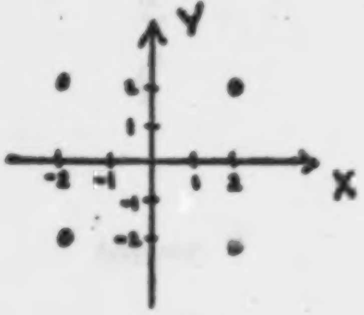
1. $A \times B$ if $A = \{1, 2\}$ and $B = \{3, -2\}$



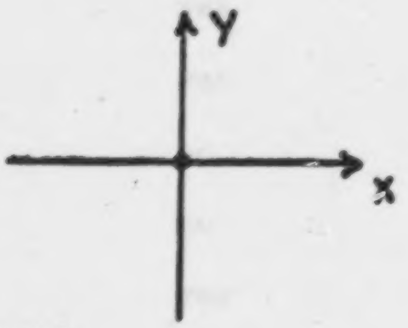
2. $B \times A$ if $A = \{1, 2\}$ and $B = \{3, -2\}$



3. $C \times C$ for $C = \{2, -2\}$



4. $D \times D$ if $D = \{0\}$



5. $(A \times B)^*$ for sets A and B in problem 1.

$B \times A$
same as problem 2.

Chapter 45: Introduction to Straight Lines

GRAPH EACH OF THE FOLLOWING BY FINDING THE x and y INTERCEPTS:

	Answer
1. $y = x + 3$	$(0, 3)$; $(-3, 0)$
2. $2y - x = 1$	$(0, \frac{1}{2})$; $(-1, 0)$
3. $3y + 2x = 12$	$(0, 4)$; $(6, 0)$
4. $4y - 3x - 18 = 0$	$(0, \frac{9}{2})$; $(-6, 0)$
5. $2x - y = 0$	$(0, 0)$ another pair of coordinates must be found.

ANSWER ∈ OR ∉ FOR EACH OF THE FOLLOWING:	Answer
1. (0, 0) ______ $\{(x, y): y = x\}$	∈
2. (-6, -9) ______ $\{(x, y): y - x = 3\}$	∉
3. (3, 2) ______ $\{(x, y): 2y - 3x = 0\}$	∉
4. (-3, 9) ______ $\{(x, y): y = x \cdot x\}$	∈
5. (0, 0) ______ $\{(x, y): y = 1/x\}$	∉

Chapter 46: Definition of a Straight Line

WHICH OF THE FOLLOWING ARE EQUATIONS OF STRAIGHT LINES:	Answer
1. $2x + 3y + 2 = 0$	yes
2. $x = - y + 1$	yes
3. $y = 3/x$	no
4. $(y/x) = x$	no
5. $y = (2/3x) - 7$	yes

UNIT 15

Supplementary Drill (Chapters 47-50)

Chapter 47: Slope of a Straight Line

FIND THE SLOPE OF THE LINE SEGMENT JOINING THE FOLLOWING POINTS:

	Answer
1. A(2, 4) B(5, 8)	4/3
2. C(3, -1) D(5, 7)	4
3. E(-2, -5) F(4, 6)	11/6
4. G(-3, 2) H(4, -7)	-9/7
5. I(7, 0) J(-4, 0)	0
6. K(-6, -2) L(-3, -1)	1/3
7. M(0, 0) N(1, -2)	-2
8. O(2, 7) P(2, 10)	not defined
9. Q(x, y) R(x , y)	$\frac{y - y}{x - x}$
10. S(a, b) T(c, d)	$\frac{d - b}{c - a}$

Chapter 48: Point Slope Equation of a Straight Line

WRITE THE POINT SLOPE EQUATION OF EACH OF THE FOLLOWING:

	Answer
1. Line A, if the slope of A = 2/3 and (1, 4) ∈ A	$(x - 1) \frac{2}{3} = y - 4$
2. Line B, if the slope of B = -3/5 and (2, -4) ∈ B	$(x - 2)(-3/5) = y + 4$
3. Line C, if the slope of C = 0 and (3, 5) ∈ C	$0 = y - 5$
4. Line D, if the slope of D = -7/6 and (-3, -2) ∈ D	$(x + 3)(-7/6) = y + 2$
5. Line E, if the slope of E = 0 and (0, 0) ∈ E	$0 = y$
6. Line F, if (2, 3) F and (4, 7) ∈ F	$(x - 2)2 = y - 3$ or $(x - 4)2 = y - 7$
7. Line G, if (-2, 3) G and (-2, -3) ∈ G	no point slope equation
8. Line H, if (-7, 0) H and (4, 0) ∈ H	$0 = y$
9. Line J, if (5, 0) J and (0, 5) ∈ J	$(x - 5)(-1) = y$ or $(x)(-1) = y - 5$

10. Line *, if $(x_i, y_i) \in *$ and $(x_j, y_j) \in *$ $\quad (x_j - x_i)\frac{y_j - y_i}{x_j - x_i} = y_j - y_i$

Chapter 49: Slope Intercept Equation of a Straight Line

(a) WRITE THE SLOPE -INTERCEPT FORM OF EACH OF THE FOLLOWING EQUATIONS.
(b) STATE THE SLOPE OF EACH.
(c) STATE THE Y-INTERCEPT OF EACH.

	Answer
1. $y - 3x = 4$	(a) $y = 3x + 4$ (b) 3 (c) 4
2. $y + 2x + 5 = 0$	(a) $y = -2x - 5$ (b) -2 (c) -5
3. $-y + \frac{1}{2}x = -3$	(a) $y = \frac{1}{2}x + 3$ (b) $\frac{1}{2}$ (c) 3
4. $3y - 2x + 4 = 0$	(a) $y = \frac{2}{3}x - \frac{4}{3}$ (b) 2/3 (c) -4/3
5. $3y + 2x = 0$	(a) $y = -2/3x$ (b) -2/3 (c) 0
6. $by - ax + c = 0$ (b 0)	(a) $y = \frac{ax}{b} - \frac{c}{b}$ (b) a/b (c) -c/b
7. $7 - x = y$	(a) $y = -x + 7$ (b) -1 (c) 7
8. $y - 7 = 0$	(a) $y = 7$ (b) 0 (c) 7
9. $\sqrt{3}y - x = 12$	(a) $y = \frac{1}{\sqrt{3}}x + 2$ (b) $\frac{1}{\sqrt{3}}$ (c) 2
10. $x - 4 = 0$	no slope intercept form since the slope is not defined for $x - 4 = 0$

Chapter 50: Systems of Equations

SOLVE THE FOLLOWING SYSTEMS OF EQUATIONS:

1.	$x + y = 2$ $x - y = 4$	$\{(3, -1)\}$	6.	$7r - 6s = 24$ $9r + 8s = -32$	$\{(0, -4)\}$
2.	$4x + y = 10$ $x + 2y = -8$	$\{(4, -6)\}$	7.	$1.2x - 2.4y = 5$ $1.2x + 2.4y = 7$	$\{(5, 1/2.4)$ or equivalent$\}$
3.	$x = 4$ $3y - 2x = 1$	$\{(4, 3)\}$	8.	$2x + y = 3$ $2x + y = 4$	$\emptyset$
4.	$y = -3$ $3x = y + 6$	$\{(1, -3)\}$	9.	$2x - 3y = 5$ $4x - 6y = 10$	$\{(x, y):\ x \in R$ and $y \in R\}$
5.	$x + 2y = 10$ $-x + 2y = 2$	$\{(4, 3)\}$	10.	$x = 3y$ $y = 1/3x + 5$	$\emptyset$

UNIT 16

Supplementary Drill (Chapter 51)

Chapter 51: Word Problems

1. The difference between one-half and one-third of a number is 30. What is the number? 180

2. Separate 50 into two parts such that four-sevenths of the larger is equal to three times the smaller. 42, 8

3. A manufacturer of cotton wishes to sell a 1000 mixture of cotton for 22¢ a pound. How many pounds of 12¢ and 40¢ cotton must he use? 357.1 lb. @$.40
642.9 lb. @ .12

4. A man has 40 gallons of a 24% nitric acid solution. How many gallons of an 80% nitric acid solution must he add in order to have a 36% solution? $10 \frac{10}{11}$ gallons

5. One boy can paint a fence in 12 days and another boy can paint the same fence in 9 days. If the two boys work together how long will it take them to paint the fence? 5 1/7 days

6. A tank has three pipes connected to it. Two of the pipes can fill the tank in 8 and 6 hours respectively. The third pipe can drain the tank in 12 hours. How long will it take to fill the tank if all three pipes are operating? 4.8 hours

7. The sum of the digits of a number is 8. The tens digit is 4 greater than the units digit. What is the number? 62

8. Two cars start 1200 miles apart and travel towards each other beginning at the same time. One car travels at the rate of 44 miles per hour and the other at 56 miles per hour. How far will each car travel before they meet? 672 miles
528 miles

9. A man received 93¢ in change. He had 2 more quarters than dimes and 5 more pennies than quarters. How many of each type of coin did he have? 3 quarters, 1 dime, 8 pennies

10. Three-hundred tickets were sold for a play. Some cost 25¢ and some cost 10¢. If $49.20 was the total box office take, how many of each price ticket did they sell? 172 10¢ tickets
128 25¢ tickets

UNIT 17

Supplementary Drill (Chapters 52-57)

MULTIPLY:

	Answer		Answer
1. $n^3 \cdot n^4$	n^7	6. $3^x \cdot 3^2$	3^{x+2}
2. $(-a^5)(a^2)$	$-a^7$	7. $(a^n)^2$	a^{2n}
3. $(a^3)(a^2)b$	a^5b	8. $x^a \cdot y^a$	$x^a \cdot y^a$
4. $3x \cdot 5x^2$	$15x^3$	9. $(a)(-a)(-a)$	a^3
5. $(-7x^2)(-6x)$	$42x^3$	10. $(-3^n)^2$	3^{2n}

EVALUATE THE FOLLOWING USING $a = 0$, $b = 1$, $c = 2$, $d = 3$, $e = 4$.

	Answer		Answer
1. b^2c	2	11. bad	0
2. $(cd)^2$	36	12. $(d^2)^c$	81
3. c^a	1	13. $(3e^2)^b$	48
4. $\frac{c^b}{a}$	not defined	14. a^{-1}	not defined
5. d^{-1}	1/3	15. d^{-c}	1/9
6. $ab^2c^3d^4$	0	16. $d^c \cdot e^c$	144
7. $\frac{(3c)^2}{b}$	36	17. $\frac{c}{e^a}$	2
8. $c^2 - d^3$	-23	18. b^{cde}	1
9. $d^3 - c^2$	23	19. b^{c+d+e}	1
10. e^d	64	20. $(4a)^d$	0

SOLVE:

	Answer		Answer
1. $2^x = 2^3$	$x = 3$	4. $3^{2p} \cdot 3^{3p} = 3^2$	$p = \frac{2}{5}$
2. $4^{t+1} = 4^{-3}$	$t = -4$	5. $6^{\frac{s}{2}} \cdot 6^{\frac{s}{4}} = 6$	$s = \frac{4}{3}$
3. $5^y \cdot 5^2 = 5$	$y = -1$	6. $7^{2m} \cdot 7^m = 1$	$m = 0$

PERFORM THE INDICATED OPERATIONS AND SIMPLIFY:

	Answer		Answer
1. $\frac{x^6}{x^2}$	x^4	6. $\frac{t^7 + t^4}{t^3}$	$t^4 + t$
2. $\frac{2^3 \cdot 2^4}{2^2}$	2^5 or 32	7. $3^{2x} \cdot 3^{2x+1}$	3^{4x+1}
3. $(2a^2)^3$	$8a^6$	8. $(1^8)^{20}$	1
4. $6^{8t} \div 6^{2t}$	6^{6t}	9. $2x^0$	2
5. $\frac{3^a}{3^b}$	3^{a-b}	10. $(2x)^0$	1

UNIT 18

Supplementary Drill

(Chapters 58-61)

Chapter 58: Polynomials

EVALUATE THE FOLLOWING POLYNOMIALS FOR $x = 2,\ y = 3,\ w = 4$	Answer
1. $x^2 + 2y - w$	6
2. $3(y^3 - w) + x$	71
3. $\frac{x + y - (w + 1)}{3}$	0
4. $x^3 - y^2 - w$	-5
5. $x(y - w)^2$	2

PERFORM THE INDICATED OPERATION:	Answer
1. $(3x^2 - 3x + 1) + (-2x^2 + x - 4)$	$x^2 - 2x - 3$
2. $(x^3 + x^2 - 5) + (2x^3 - 4x^2 - 4x + 7)$	$3x^3 - 3x^2 - 4x + 2$
3. $(x^2 - 3x + 3) - (x^2 - 2x + 5)$	$-x - 2$
4. $(3x^3 + 7x^2 - 7) - (-4x^2 + 3x + 1)$	$3x^3 + 11x^2 - 3x - 8$
5. $(x^4 - x^3 + x^2 - x + 1) - (x^4 - x^3 + x^2 - x + 1)$	0

Chapter 59: Multiplication of Monomials

FIND THE PRODUCT:	Answer		Answer
1. $(8x^2)(8x^2)$	$64x^4$	6. $-3t^2(x^2 - 4y^3)$	$-3t^2x^2 + 12t^2y^3$
2. $(3y^4)(-10y^2)$	$-30y^6$	7. $18r^7(\frac{1}{6}r^3 - \frac{1}{3}r^7 + \frac{1}{9}r)$	$3r^{10} - 6r^{14} + 2r^8$
3. $(2x^0)(3t^2)$	$6t^2$	8. $(-xy^3)(-x^3y)$	x^4y^4
4. $(2ab)(\frac{1}{4}a^2b^2)$	$\frac{1}{2}a^3b^3$	9. $(t^2 - t^3)t^0$	$t^2 - t^3$
5. $x^2y(y^3x^4)w$	x^6y^4w	10. $bd(-a^3 + bd - abc + d^4)$	$-a^3bd + b^2d^2 - ab^2cd + bd^5$

Chapter 60: Multiplication of Polynomials

MULTIPLY AND COLLECT LIKE TERMS:

		Answer			Answer
1.	$\begin{array}{l} x + 2 \\ \underline{x + 2} \end{array}$	$x^2 + 4x + 4$	6.	$\begin{array}{l} 4a^2 - a + 10 \\ \underline{5a - 6} \end{array}$	$20a^3 - 29a^2 + 56a - 60$
2.	$\begin{array}{l} y + 3 \\ \underline{y - 3} \end{array}$	$y^2 - 9$	7.	$\begin{array}{l} 3x^2 - 4x - 8 \\ \underline{x^2 + 2x - 1} \end{array}$	$3x^4 + 2x^3 - 16x^2 - 12x + 8$
3.	$\begin{array}{l} 3x + 2 \\ \underline{2x - 6} \end{array}$	$6x^2 - 14x - 12$	8.	$\begin{array}{l} 2y - y^2 + 4 \\ \underline{8 - 3y} \end{array}$	$y^3 - 14y^2 + 4y + 32$
4.	$\begin{array}{l} y - 1 \\ \underline{y - 1} \end{array}$	$y^2 - 2y + 1$	9.	$\begin{array}{l} x^2 - 1 + 2x \\ \underline{3 - 2x^2} \end{array}$	$-2x^4 - 4x^3 + 5x^2 + 6x - 3$
5.	$\begin{array}{l} x^2 - 2 \\ \underline{x^2 - 1} \end{array}$	$x^4 - 3x^2 + 2$	10.	$\begin{array}{l} a + b + c \\ \underline{a + b + c} \end{array}$	$a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

Chapter 61: Division of Polynomials

DIVIDE:

		Answer
1.	$\dfrac{xyz + xy}{x}$	$yz + y$
2.	$\dfrac{abc - 3ab}{ab}$	$c - 3$
3.	$\dfrac{2a^3 - a^2 - ab}{a}$	$2a^2 - a - b$
4.	$\dfrac{4x^4 - 2x^3 + 16x}{2x}$	$2x^3 - x^2 + 8$
5.	$\dfrac{8x^5 - 10x^4 + x^3}{x^3}$	$8x^2 - 10x + 1$
6.	$\dfrac{2b^8 + 4b^9 + 2b^{10}}{2b^7}$	$b + 2b^2 + b^3$
7.	$\dfrac{9a^{10} - 9a^9}{9a}$	$a^9 - a^8$
8.	$\dfrac{-5x - 30x^{10}}{5x}$	$-1 - 6x^9$
9.	$\dfrac{18a^3 + 12a^2 - 9a}{-3a}$	$-6a^2 - 4a + 3$
10.	$\dfrac{4x^3y^2z - 14x^2yz^5}{2xyz}$	$2x^2y - 7xz^4$

UNIT 19

Supplementary Drill (Chapters 62-65)

Chapter 62: Factoring

FACTOR:		Answer
1.	$a^2 - b^2$	$(a + b)(a - b)$
2.	$5x^4 - 45x^2$	$5x\ (x + 3)(x - 3)$
3.	$2x^2 + 7x + 3$	$(2x + 1)(x + 3)$
4.	$t^2 + 2t - 35$	$(t + 7)(t - 5)$
5.	$7p^2 - 112q^2$	$7(p + 4q)(p - 4q)$
6.	$18s^2t - 60\ st + 50t$	$2t(3s - 5)(3s - 5)$
7.	$72 - m - m^2$	$(9 + m)(8 - m)$
8.	$3y^6 - 78y^4 + 75y^2$	$3y^2(y + 5)(y - 5)(y + 1)(y - 1)$
9.	$75 - 147x^2$	$3(5 + 7x)(5 - 7x)$
10.	$rx^8 - ry^8$	$r(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

Chapter 63: Rational Expressions

SIMPLIFY WHENEVER POSSIBLE:

		Answer			Answer
1.	$\frac{2t - 3s}{4(2t - 3s)}$	$\frac{1}{4}$	6.	$\frac{ax + by}{ax - by}$	simplified
2.	$\frac{6a + 6b}{a^2 - b^2}$	$\frac{6}{a - b}$	7.	$\frac{2x^3 - 2x^2 + 4x}{3x^3 + 3x^2 - 18x}$	$\frac{2(x + 1)}{3(x + 3)}$
3.	$\frac{x^2 - 5x + 6}{x^2 - 4x + 4}$	$\frac{x - 3}{x - 2}$	8.	$\frac{(a^2 + 5a)(a^2 - 4a)}{(a^2 - 16)(a^2 - 25)}$	$\frac{a^2}{(a + 4)(a - 5)}$
4:	$\frac{3m + 21}{m^2 - 49}$	$\frac{3}{m - 7}$	9.	$\frac{6a^2 + a - 15}{6a^2 - 13a + 6}$	$\frac{3a + 5}{3a - 5}$
5:	$\frac{x^2 + 5x - 4}{x - 4}$	simplified	10.	$\frac{(m - 1)^3}{(1 - m)^2}$	$m - 1$

Chapter 64: Radicals

SIMPLIFY WHENEVER POSSIBLE: U = reals

	Answer		Answer		Answer
1. $\sqrt{9}$	3	6. $\pm\sqrt{4}$	± 2	11. $\sqrt{(-3)^2}$	3
2. $\sqrt{x^2}$	$\lvert x\rvert$	7. $-\sqrt{16}$	-4	12. $\sqrt{7}$	can't be simplified
3. $\sqrt{(-x)^2}$	$\lvert x\rvert$	8. $\sqrt[3]{64}$	4	13. $\sqrt{-\frac{1}{4}}$	not real
4. $\sqrt{9a^2}$	$3\lvert a\rvert$	9. $\sqrt[3]{-64}$	-4	14. $\sqrt[3]{125}$	5
5. $\sqrt{\frac{1}{4}}$	$\frac{1}{2}$	10. $\sqrt{-64}$	not real	15. $(\sqrt[4]{86})^4$	86

Chapter 65: Simplifying Radicals

IF a, b, n, and m REPRESENT POSITIVE NUMBERS, WHICH OF THE FOLLOWING ARE ALWAYS TRUE?

	Answer		Answer
1. $\sqrt{a}\cdot\sqrt{b} = ab$	yes	6. $(a^m)^n = a^{m+n}$	no
2. $\sqrt{a}+\sqrt{b}=\sqrt{a+b}$	no	7. $(\sqrt[m]{a})^n = a^{n/m}$	yes
3. $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$	yes	8. $\sqrt[m]{b^m} = b$	yes
4. $(ab)^m = a^m b^m$	yes	9. $a^m \cdot b^n = (ab)^{m+n}$	no
5. $(a+b)^m = a^m + b^m$	no	10. $\sqrt{a}-\sqrt{b}=\sqrt{a-b}$	no

SIMPLIFY WHENEVER POSSIBLE:

1. $\sqrt{\frac{1}{2}}$	$\frac{\sqrt{2}}{2}$	6. $\sqrt{27}-\sqrt{75}+\sqrt{48}$	$2\sqrt{3}$
2. $\sqrt{18}$	$3\sqrt{2}$	7. $\sqrt{9a^3b^4}$	$3\lvert a\rvert b^2\sqrt{a}$
3. $\sqrt{-18}$	$3\sqrt{-2}$	8. $\sqrt{\frac{5}{8}}$	$\frac{\sqrt{10}}{4}$
4. $\sqrt[3]{16}$	$2\sqrt[3]{2}$	9. $\frac{3}{\sqrt{11}}$	$\frac{3\sqrt{11}}{11}$
5. $3\sqrt{2}+7\sqrt{2}-10\sqrt{2}$	0	10. $\sqrt{3}-\sqrt{2}$	$\sqrt{3}-\sqrt{2}$

UNIT 20

Supplementary Drill (Chapters 66-69)

Chapter 66: Radical Equations

SOLVE AND CHECK YOUR ANSWERS:

	Answer		Answer
1. $\sqrt{x} = 4$	$\{16\}$	6. $2\sqrt{5x} = 6$	$\{9/5\}$
2. $\sqrt{t - 1} = -3$	$\emptyset$	7. $\sqrt{x} = \frac{\sqrt{3}}{2}$	$\{3/4\}$
3. $\sqrt{m^2 - 3} = \frac{1}{2}$	$\left\{\frac{\sqrt{11}}{2}, -\frac{\sqrt{11}}{2}\right\}$	8. $3\sqrt{x - 9} = 2\sqrt{x + 11}$	$\{25\}$
4. $\sqrt{x + 2} - 3 = 5$	$\{62\}$	9. $5\sqrt{x - 2} - 3\sqrt{2x + 3} = 0$	$\{11\}$
5. $\sqrt{x} + \sqrt{3} = 2$	$\{7 - 4\sqrt{3}\}$	10. $\sqrt[3]{x - 2} = 2$	$\{10\}$

Chapter 67: Quadratic Equations

SOLVE:

	Answer		Answer
1. $t^2 - t - 6 = 0$	$\{3, -2\}$	6. $5s + s^2 = 0$	$\{0, -5\}$
2. $m^2 - 9m + 20 = 0$	$\{5, 4\}$	7. $-3x^2 - x + 10 = 0$	$\{5/3, -2\}$
3. $p^2 = 9p - 14$	$\{7, 2\}$	8. $(x - 1)(x + 4) = 6$	$\{2, -5\}$
4. $x^2 - 24 = 5x$	$\{8, -3\}$	9. $2t^2 - 4t - 30 = 0$	$\{5, -3\}$
5. $4x^2 - 25 = 0$	$\{5/2, -5/2\}$	10. $(2x + 1)(3x + 7) = (6x + 3)(2x - 5)$	$\{-1/2, 22/3\}$

Chapter 68: Completing the Square

SOLVE THE FOLLOWING BY COMPLETING THE SQUARE:

	Answer
1. $x^2 + 5x - 6 = 0$	$\{-6, 1\}$
2. $m^2 - 2m - 7 = 0$	$\{1 + 2\sqrt{2}, 1 - 2\sqrt{2}\}$
3. $5t^2 + 7t - 6 = 0$	$\{-2, +3/5\}$
4. $6y^2 + y - 1$	$\{-1/2, 1/3\}$
5. $7s^2 + 9s = +10$	$\{-2, +5/7\}$
6. $3s^2 - 7s + 4 = 0$	$\{4/3, 1\}$
7. $2p^2 + 2p - 1 = 0$	$\left\{\frac{-1 + \sqrt{3}}{2}, \frac{-1 - \sqrt{3}}{2}\right\}$

8. $12v^2 - 7v + 1 = 0$ $\{1/4, 1/3\}$

9. $15q^2 + 29q - 14 = 0$ $\{2/5, -7/3\}$

10. $dw^2 + ew + f = 0 \quad (d \neq 0)$ $\frac{-e+\sqrt{e^2-4df}}{2d}, \frac{-e-\sqrt{e^2-4df}}{2d}$

Chapter 69: Quadratic Formula

SOLVE USING THE QUADRATIC FORMULA:

1. $x^2 + 3x + 2 = 0$ $-1, -2$
2. $y^2 - 9y + 5 = 0$ $\frac{9+\sqrt{61}}{2}, \frac{9-\sqrt{61}}{2}$
3. $2t^2 + t - 3 = 0$ $1, -3/2$
4. $3x^2 = 12 - 5x$ $4/3, -3$
5. $3 - 4z + z^2 = 0$ $3, 1$
6. $3x^2 - 4 = 0$ $\frac{2\sqrt{3}}{3}, \frac{-2\sqrt{3}}{3}$
7. $3x \ - 5x = 0$ $0, 5/3$
8. $8m = m \ + 2$ $4\ \sqrt{14}, 4 - \sqrt{14}$
9. $(2x + 1)(x - 3) = 5x$ $\frac{5+\sqrt{31}}{2}, \frac{5-\sqrt{31}}{2}$
10. $\frac{x}{2x-1} = \frac{x-1}{3x-1}$ $-1+\sqrt{2}, -1-\sqrt{2}$

UNIT 21

Supplementary Drill (Chapters 70-71)

Chapter 70: Solution by Graphing

IF THE UNIVERSE IS R, FIND THE APPROXIMATE ROOTS OF EACH OF THE FOLLOWING BY GRAPHING.

1. $x^2 + x - 6 = 0$
2. $x^2 - 4x + 4 = 0$
3. $x^2 - 3x + 5 = 0$
4. $2x^2 - 5x - 3 = 0$
5. $3x^2 - x - 5 = 0$

Chapter 71: Quadratic Equation Word Problems

1. If the product of two consecutive integers is 210, find the integers.
   14, 15

2. The square of a number is 56 more than the number itself. What is the number?
   8, -7

3. The perimeter of a rectangle is 30 inches and its area is 54 square inches. What are its dimensions?
   length = 6 in.
   width = 9 in.

4. Two men can do a job together in 6 hours. The first man can do the job alone in 5 hours less time than the second man working alone. How long does it take each man to do the job alone?
   first man 10 hrs.
   second man 15 hrs.

5. A man, using a motorboat, traveled up the coast for a distance of 40 miles. On the return trip his speed was 2 m.p.h. faster, and the time required was one hour less. What was the boats speed up the coast?
   8 m.p.h.

6. The difference in the squares of two consecutive odd integers is 56. What are the integers?
   13, 15

7. A class of 24 students is seated in rows, with each row containing the same number of seats. If there were two less seats in each row, one more would be needed. How many seats are in each row?
   8

8. The side of one square is 3 feet greater than the side of a second square. Twice the area of the smaller square is 103 square feet greater than the area of the larger square Find the side of each square.
   14 ft. 17 ft.

9. Three boys John, Ralph, and Sidney can polish a car together in 1 hour and twenty minutes. To polish the car alone, Ralph would take twice as long as John and 2 hours longer than Sidney. How long would it take each to do the job alone? John 3hrs., Ralph 4 hrs., Sidney 6 hrs.

10. A man drove 200 miles at a certain rate, and then he drove 100 miles at a rate 10 m.p.h. slower. Had he driven the entire distance at the faster rate, his time for the entire trip would have been $\frac{1}{2}$ hour less. Find his original rate. 50 m.p.h.

UNIT 22

Supplementary Drill (Chapters 72-73)

Chapter 72: Solution Sets of Inequalities

SOLVE THE FOLLOWING INEQUALITIES FOR y:

1. $y + 2 \leq x$	$\{(x, y): y \leq x - 2\}$
2. $y - x - 3 > 0$	$\{(x, y): y > x + 3\}$
3. $-y + x^2 \leq x - 3$	$\{(x, y): y \geq x^2 - x + 3\}$
4. $3y - 2x > 4 - y$	$\{(x, y): y > \frac{1}{2}x + 2\}$
5. $-2/5y + 3x - 1 \leq x - 2$	$\{(x, y): y \geq 5x + 5/2\}$

GRAPH THE FOLLOWING SYSTEMS OF INEQUALITIES:

1. $y < 2x$
   $x > 1$

2. $y \geq 3x - 6$
   $y \leq 2x + 4$

3. $y \geq x$
   $x \geq 0$
   $y \leq 4$

4. $y > 2x - 1$
   $x + y > 2$
   $x > 0$

5. $y \leq 2x - 3$
   $x > 0$
   $y \geq 0$

Chapter 73: Proof

This chapter does not readily lend itself to drill work, however, additional work may be found in the supplementary enrichment section of this manual.

SUPPLEMENTARY ENRICHMENT

Biographies *

1. **Thales**
2. **Pythagoras**
3. **Zeno**
4. **Aristotle**
5. **Euclid**
6. **Eratosthenes**
7. **Archimedes**
8. **Hypatia**
9. **Fibonacci**
10. **Tartaglia**
11. **Cardano**
12. **Fermat**
13. **Banneker**
14. **Descartes**
15. **Pascal**
16. **Newton**
17. **Leibniz**
18. **Euler**
19. **Gauss**
20. **Abel**
21. **Boole**
22. **Poincaré**
23. **Einstein**
24. **Dedekind**
25. **Dase**

*** For source materials see Biography section of the Bibliography.**

Selected Topics

* References correspond to numbered bibliography.

1. Abacus

* History - 2, 4. Special Topics - 2, 41, 63.

2. Boolean Algebra

Modern Mathematics - 3, 19, 29.

3. Calculating Prodigies

Special Topics - 5, 6, 7, 54, 62. Mathematical Recreations - 3.

4. Calculating Rapidly

Special Topics - 29(7), 65.

5. Casting Out Nines

Mathematical Recreations - 1, 13.

6. Computers

Special Topics - 13, 14, 15, 23, 51(1), 72, 76(1).

7. Fermat's Last Theorem

History - 3. Special Topics - 26. Mathematical Recreations - 3.

8. Fibonacci Numbers

History - 3. Special Topics - 9, 16, 51(14). Mathematical Recreations -13.

9. Fields (Algebraic)

Modern Mathematics - 19, 21. Special Topics - 39.

10. Four Color Problem

Special Topics - 4, 27. Mathematical Recreations - 15.

11. Golden Section

Traditional Mathematics - 14. Special Topics - 11.

12. Graphs

Special Topics - 29(1), 50, 51(7).

13. Group Theory

    Traditional Mathematics - 8. Special Topics - 11, 21.

14. Inequalities

    Special Topics - 10, 18, 45.

15. Infinity

    Popular Expositions - 12. Modern Mathematics - 11. Special Topics - 52, 76(2), 77.

16. Large Numbers Including Googol and Googolplex

    Popular Expositions - 5. Special Topics - 20, 22.

17. Linear Programming

    Modern Mathematics - 15. Special Topics - 40, 51(16).

18. Magic Squares

    Mathematical Recreations - 3, 10, 13.

19. Mathematical Fallacies

    Popular Expositions - 5. Mathematical Recreations - 3.

20. Möbius Strips

    Special Topics - 31, 71.

21. Napier Rods (Bones)

    History - 2, 3.

22. Nim - A Binary Game

    Traditional Mathematics - 8. Mathematical Recreations - 3, 7.

23. Number Systems

    Popular Expositions - 2. History - 3. Special Topics - 29(12).
    Mathematical Recreations - 1, 13.

24. Number Tricks

    Popular Expositions - 2. Mathematical Recreations - 1, 4, 22.

25. Paper Folding (Origami).

    Mathematical Recreations - 11, 18, 24.

26. Pi

    Special Topics - 8, 44, 57.

27. Prime Numbers

    Special Topics - 42, 43, 47.

28. Pythagorean Theorem

    History - 3. Special Topics - 12, 17, 29(8), 32.

29. Rhind Papyrus

    History - 1, 3. Special Topics - 28.

30. Sets

    Modern Mathematics - 7. Special Topics - 24, 29(6), 36, 46, 68, 73.

31. Topology - Its Unusual Applications

    Popular Expositions - 5, 12. Special Topics - 11, 29(11), 70.

32. Towers of Hanoi and Chinese Rings

    Popular Expositions - 5. Mathematical Recreations - 1, 3.

33. Truth Tables

    Popular Expositions - 3. Traditional Mathematics - 8. Modern Mathematics - 20, 25.

34. Zeno's Paradoxes

    History - 3. Biography - 3. Popular Expositions - 5. Mathematical Recreations - 15.

35. Zero - Its Invention and Usuage

    Popular Expositions - 6. Special Topics - 11, 25, 52.

Suggested Activities

1. Invent new symbols and a new numeration system.
2. Construct and demonstrate how to use a transit.
3. Devise an original field, group, or ring.
4. Solve an original problem using linear programming
5. Invent a new system of measurement with appropriate units and measuring devices.
6. A study of ancient number systems.
7. Calculation of pi.
8. Investigate potential careers in mathematics.
9. A study of some elementary concepts of probability.
10. Write an original mathematical poem, essay, or story.

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    8. HYPERBOLIC FUNCTIONS (Sheratov, V. G.)
    9. INDUCTION IN GEOMETRY (Golovina, L. I., and Yaglom, I. M.)
    10. MISTAKES IN GEOMETRIC PROOFS (Dubnov, Ya. S.)
    11. PROOF IN GEOMETRY (Fetisov, A. I.)
    12. SIMPLEST MAXIMA AND MINIMA PROBLEMS (Natanson, I. P.)
    13.. SUMMATION OF INFINITELY SMALL QUANTITIES (Natanson, I. P.)

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Book V Unit VII Radicals
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Doubleday and Co., Inc., Garden City, Long Island, New York.
Encyclopaedia Britannica Films, Inc., Wilmette, Ill., Attn: TEMAC
New York University of Technology, New York, New York
SRA Materials of instruction, Science Research Associates, Inc., 259 East Erie Street, Chicago, 11, Illinois.
Teaching Machines, Inc., 235 San Pedra Drive, N.E. Albuquerque, New Mexico

Numbers	Subject	Producer	Type of Response		Branching	Correct Response-Advancement	Consumable Materials	Audio-Visual	Cost	No. of Frames	Frame Exposures Single	Available 9/1/1963	Type of Course		Grade Level
			Mult. Choice	Const.									Basic	Supplement	
1.	Algebra (Adventures In)	Doubleday Tutortext	X		X	X		X	3.95	350	X	X	X		8-10
2.	Algebra (Introduction To)	Doubleday Tutorfilm	X		X	X	Film	X	35.00	350	X	X			8 up
3.	Algebra I	Teaching Machines		X			X		7.50	2000	X	X	X	X	
4.	Algebra I	Program Teaching Aids		X	X						X	X	X		8-10
5.	Algebra I	E.B.F.		X		X			12.75 sp.50	8200	X	X	X		H.Sh.
6.	Slide Rule (use of)	Doubleday Tutortext	X		X	X		X	3.95	400	X	X	X	X	9 up
7.	Algebra	N. Y. Institute of Technology			X			X		10000		X			C
8.	Algebra	National Teaching Machines		X			X	X	5.50	2,000	X		X	X	C./Hs
9.	Algebraic Structures Groups and Fields	Bucknell		X		X				1,650		X	X	X	12
10.	Fields and Ordered Fields	E.B.F.		X		X			7.50	1,828	X	X	X		C
11.	Set, Relations and Functions	Bucknell		X		X				1,050			X	X	12
12.	Logarithms	New York Inst. of Technology			X			X		600		X			A
13.	Modern Mathematics	Science Research		X					10.00	8,400	X	X	X		9or8
14.	Modern Mathematics	Teaching Mach.		X			X				X	X	X	X	
15.	Computer Mathematics	Teaching Mach.		X			X				X	X	X	X	
16.	Binary Numbers	New York Inst. of Technology			X			X		600		X			A

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ADDITION AND SUBTRACTION OF RATIONAL NUMBERS , Traditional, 31 minutes. Modern Learning Aids, Summit, New Jersey. Rental $6.00 each for 3 school days. Begins with a brief review of Film #3201, Natural Numbers, Integers and Rational Numbers, and leads into a discussion of the properties of rational numbers, emphasizing the conditions under which definitions are not valid. Includes magnitudes, negative numbers, the idea of the absolute value, and the sums, and differences of positive and negative numbers.

ADVENTURES IN NUMBER AND SPACE, Modern, Association Films, Inc., Ridgefield, New Jersey. Rental $7.50. Dr. Howard Fehr, and Bill Baird Marionettes.

ALGEBRA IN EVERYDAY LIFE, Traditional, 11 minutes. Coronet Films, Chicago 1, Illinois. Rental $7.00, Indiana University. Presents the nature of algebra and its practical applications in an everyday situation, thus motivating interest in its study.

ALGEBRAIC AND COMPLEX FRACTIONS, Traditional, 29 minutes. Modern Learning Aids, Summit, New Jersey. Rental $6.00 per 3 days. The significance of fractional notation for algebraic symbols is reviewed. The product of two fractions, the reciprocial of a reciprocal, and the invert-and multiply rule are all presented. The addition, and subtraction of algebraic fractions is shown by using a common denominator. Also simplifying.

BASE AND PLACE, Modern, 30 minutes. Association Film, Inc., Ridgefield, New Jersey. Rental $5.75 Pennsylvania State University Audio-Visual Library. Presents the characteristics, history, and applications of the binary system. (digital computers)

BIG NUMBERS, Traditional, 31 minutes. Association Films, Inc., LaGrange, Illinois. Rental $5.75 Pennsylvania State University Audio-Visual Liabrary. Demonstrates how scientists, and mathematicians write, and use very large, and very small numbers. Stress the meaning, and importance of exponents, and shows some of their uses.

CAREERS IN MATHEMATICS, Traditional, 30 minutes. Pennsylvania State University Audio-Visual Library. Rental $5.75. Recap of eight films in the series. Rewards of a career in mathematics.

* CONCEPT OF FUNCTION, Modern, 16 minutes. McGraw-Hill, New York. Dr. Albert E. Meder, Jr., narrator. Rental $4.75, Pennsylvania State University Audio-Visual Library. A new approach to the concept of "function" which eliminated the former ambiguity, and presents new and clearer definitions of such terms as: F(x), range domain, and rule. Confusion between sinle-valued, and multiple-valued functions is eliminated. * These films are available without cost from the Department of Public Instruction.

DEVELOPING AND SOLVING LINEAR EQUATIONS, Traditional, 30 minutes. Modern Learning Aids, Summit, New Jersey. Rental $6.00 per 3 days. The proposition is first made that x stands for a number, with no restrictions. Slowly the need for the establishment of certain consistent restrictions on x is shown. The general solution of any linear equation is discussed, and the importance of checking solutions is stressed.

DONALD IN MATH-MAGIC LAND, Traditional. Association Films, Inc., LaGrange, Illinois. (Walt Disney film)

THE EARLIEST NUMBERS, Traditional, 30 minutes. Association Film, Inc. Rental $5.75, Pennsylvania State University Audio-Visual Library. Stresses man's first efforts to count in terms of the essential elements of a modern numeration system.

FIVE FUNDAMENTAL POSTULATES FOR ALGEBRA, Modern, 30 minutes. Modern Learning Aids, Summit, New Jersey. Rental $6.00 per 3 days. Shows that the rules are rules and not tested theorems. Covers the five postolates. The meaning of division, and the exclusion of division by zero are covered. The addition of fractions, and the meaning of the least common multiple and the lowest common denominator are reviewed.

FUNDAMENTAL OPERATIONS, Modern, 30 minutes. Association Films, Inc. Rental $5.75, Pennsylvania State University Audio-Visual Library. Analyzes modular arithmetic. Demonstrates that the two fundamental operations are addition, and multiplication: that subtraction, and division are the inverse of these operations.

HISTORICAL INTRODUCTION TO ALGEBRA, Modern, 27 minutes. Modern Learning Aids. Rental $6.00 per 3 days. Growth of mathematical knowledge. Also questions dealing with recent times.

HOW MAN LEARNED TO COUNT, Traditional. Association Films, Inc. Rental $7.50. A trip back through history to see how arithmetic developed.

HOW TO FIND THE ANSWER, Traditional, 11 minutes. Coronet Films. Rental Indiana University. Stresses the importance of applying the correct mathematical procedure to an appropriate problem and shows how this is done.

INTRODUCTION TO FACTORING, Modern, 30 minutes. Modern Learning Aids. Rental $6.00. The concept of factoring algebraic expression is introduced. Methods used in arithmetic are applied to examples in algebra. Warns about concelling across add or subtract signs. Factoring forms are shown for Monomial common term, Difference of two squares, Quadratic Perfect square.

INTRODUCTION TO QUADRATIC EQUATIONS, Modern, 30 minutes. Modern Learning Aids. Rental $6.00 per 3 days. The solution of word problems leading to quadratic equations is worked out in detail, including the use of the method of completing the square. The Quadratic Formula is derived and used to solve an earlier problem discussed in the film.

* IRRATIONAL NUMBERS, Modern, 23 minutes. McGraw-Hill, 1959. Rental $4.75, Pennsylvania State University Audio-Visual Library. Dr. Albert E. Meder, narrator. A demonstration for teaching irrational numbers through the study of decimal fractions. The limited understanding in past teaching is eliminated by this new method which leads to a positive demonstration of what an irrational number is. * These films are available without cost from the Department of Public Instruction.

IT'S ALL ARRANGED, Traditional, 30 minutes. Pennsylvania State University Audio-Visual Library, rental $5.75. Bill Baird and his Marionettes figure combinations mathematically.

LANGUAGE OF GRAPHS, Traditional. Coronet Films. Rental Indiana University.

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LINEAR EQUATIONS IN ONE UNKNOWN, Traditional, 30 minutes. Modern Learning Aids. Rental $6.00. Two word problems are solved in detail. The first is finding the "point of no return" in an airplane flight, and the second involves computing the amount of alcohol needed to produce a certain percentage of mixture in an automobile radiator. Film ends with a discusion of equivalent equations and extraneous roots.

MATHEMATICIAN AND THE RIVER, Traditional, 20 minutes. Pennsylvania State University Audio-Visual Library. Rental $7.00. Problems in applied science on the Mississippi River; shows the relationship between "abstract world of mathematics, and the real world of mature."

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MULTIPLICATION OF RATIONAL NUMBERS, Traditional, 31 minutes. Modern Learning Aids. Completes the review of the rational system and the fundamental operations of algebra. The multiplication, and division of signed numbers is developed carefully. Applications of these concepts are carried through the addition, subtraction, multiplication, and division of multiple term expressions involving powers.

MYSTERIOUS "X", Traditional, 30 minutes. Association Films, 1958. Rental Pensylvania State University Audio-Visual Library, $5.75. Everyday applications of algebra.

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RATIO AND PROPORTION, Traditional, 11 minutes. Coronet Junior High. Rental Indiana University.

* SENTENCES AND SOLUTION SETS, Modern, 21 minutes. McGraw-Hill. Dr. Albert E. Meder, Jr., narrator. Rental $4.25 Pennsylvania State University Audio-Visual Library. An explanation of how to introduce sets, variables, and open sentences, using actual classroom scenes, showing how the concept of set makes the teaching of inequalities and equations simpler and clearer.

SETS, CROWS, AND INFINITY, Modern, 12 minutes. Film Association of California, Los Angeles. The film explores the mathematical ideas of sets and one-to-one correspondence between two sets. An infinite set is shown to be a set which is equivalent to the infinite set of whole numbers.

THE SLIDE RULE: "C" AND "D" SCALE, Traditional, 22 minutes. Pennsylvania State University Audio-Visual Library. Rental $4.75. THE SLIDE RULE: PROPORTION, PERCENTAGE, SQUARES, AND SQUARE ROOTS, 21 minutes. Rental $4.25.

MORE SOLUTIONS OF LINEAR EQUATIONS, Traditional, 30 minutes. Modern Learning Aids. Rental $6.00. Good strategy in problem solving is developed and examples are given. The example problems are in: 1. Relations in the rational number system, 2. Mixture. 3. In uniform motion. 4. Velocities. Complete solutions are obtained and checked.

In the Film Reference section of the Supplemental Manual films marked * are available without cost from the Department of Public Instruction.

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